A Valuation Model of Mortgage Insurance Premiums Considering the Target Prescribed Capital Requirement for Systematic Risk

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Abstract

This study derives a closed-form formula for mortgage insurance (MI) premium that includes the systematic and idiosyncratic risks based on option-pricing theory. This formula can be used to calculate the MI premium considering the target prescribed capital requirement, which aims to manage the systematic risk at a given confidence level. Thus, using it can help insurers against huge changes in the economic environment. We use U.S. market data to illustrate the application of our model. Our results show that the idiosyncratic risk is a larger proportion of the total housing risk than is the systematic risk; and MI premiums based on considering both systematic and idiosyncratic risks are higher than without considering these two risks. Moreover, the sensitivity analyses reveal that the MI premium is positively related to the parameters of both the systematic and unsystematic risks. Our pricing formula for an MI premium and the results of numerical analyses should make it easier for MI insurers to account for the economic environment in determining fair MI premiums and to effectively undertake sophisticated risk management.

Keywords: Systematic Risk, Idiosyncratic Risk, Mortgage Insurance, Default, Valuation

JEL: G1, G21, G22

1. Introduction

In 2007, a sub-prime mortgage crisis caused worldwide financial market turbulence. This crisis produced large losses given default for mortgage insurance institutions. Since 2007, the mortgage insurance industry has paid more than $50 billion in claims to lenders and investors. The members of the MICA (Mortgage Insurance Companies of America) collectively had a sharp increase in their average loss ratio from 36.23% in 2005 to 218.41% in 2008. Moreover, the combined ratio also increased from 60.44% in 2005 to 237.84% in 2008.[1] As is well known, mortgage insurance (MI) system plays an important role in the housing finance market because it provides protection to lenders against losses associated with mortgage defaults and facilitates the creation of secondary mortgage markets. The MI premium is the main revenue for MI institutions. For maintaining a good MI system, MI institutions need an appropriate valuation model for accurately estimating the MI premium. The 2007 financial crisis is usually treated as a kind of systematic risk in valuation theory. Thus, considering systematic risk poses new challenges to the pricing of MI premiums. For the purpose of accurately pricing MI premiums, the main purpose of this study was to support an MI valuation model including the systematic risks of the housing market.

Recent trends indicate that setting the target PCR (prescribed capital requirement) at 99.5% as a value at risk (VaR) measure over a 1-year time frame is emerging as the international standard.[2] To meet the target PCR of the solvency rules, insurers must

[1] For more detailed information, please see the website: https://fcic-static.law.stanford.edu/cdn_media/fcic-docs/0000-00-00%20Mortgage%20Insurance%20Companies%20of%20America,%202009-2010%20Fact%20Book%20and%20Member%20Directory.pdf.

[2] For example, according to Solvency II, the SCR (solvency capital requirement) is based on a VaR measure calibrated to the 99.5% confidence level over a 1-year time horizon. Furthermore, the solvency requirements of Solvency II are economy risk-based. They are more risk-sensitive and more sophisticated than in the past, thus enabling better coverage of the real risks run by any particular insurer.

obey the solvency capital requirement. Mortgage insurers must operate within a 25-to-1 ratio of risk to capital, which means they must set aside $1 of capital for every $25 of risk they incur.

This study provides another method for MI insurers to meet the target PCR of the solvency rules. In general, the mortgagors pay the MI premium to MI companies (insurers). If measuring the MI premiums does not consider the target PCR of systematic risk, MI insurers may bear extra costs caused from the worst situation of systematic risk for obeying the solvency capital requirement, as justly mentioned. However, if mortgage insurers valuate the MI premium has considered the target PCR of the systematic risk, they can receive the premium including the potential losses caused from the worst situation of systematic risk. Then they can reduce prepayment of the solvency capital requirement and effectively allocate their capital. Accordingly, it is important for MI companies to adopt an MI valuation model which considers the target PCR for the systematic risk. The current MI valuation model does not consider the target PCR of the systematic risk. Thus, this study tries to look at it from another angle and supports a suitable MI valuation model that considers such situation.

Options-based pricing models have usually been the choice for pricing and analyzing MI contracts (Schwartz and Torous, 1992; Kau, Keenan and Muller, 1993; Lai and Gendron, 1994; Kau and Keenan, 1996, 1999; Azevedo-Pereira, Newton and Paxson, 2002, 2003; Bardhan, Karapandza and Urosevic, 2006). Nowadays, a famous options pricing theory, the martingale pricing method, is commonly used to evaluate the contracts with the options-style payoffs. This study also uses the martingale pricing method to value the MI contract. Using this method, accurate pricing of MI premiums depends mainly on a reasonable specification of the probability of default

(PD) and of the loss given default (LGD) of a mortgage. Much research has shown that changes in the housing price significantly influence the PD and LGD of a mortgage. (e.g., Schwartz and Torous, 1989, 1993; Quigley and Van Order, 1990, 1995; Smith et al., 1996; Hurt and Felsovalyi, 1998; Frye, 2000a, b; 2003; Jokivuolle and Peura, 2003; Lambrecht, et al., 2003; Archarya et al., 2004; Dermine and Carvalho, 2006).

Hendershott and Van Order (1987) find evidence that MI premiums are not very sensitive to interest rate volatility. Thus, a number of authors assume that the interest rate is a deterministic variable and let the housing risk (i.e., the house price volatility) become the most important factor for obtaining a closed-form formula for MI premiums when using options-based pricing models (Bardhan et al., 2006; Chen et al., 2010; Chang et al., 2012; Pu et al., 2016). To derive a closed-form formula for an MI premium based on an options-based pricing model, we also follow their specifications and assume that the housing risk is the most important factor.[3]

The housing risk can be divided into two parts: the housing systematic risk and the housing idiosyncratic risk (or non-systematic risk). As for the systematic risk in the housing market, a large number of studies have investigated the significant effects of macroeconomic factors on house prices or returns (Case and Shiller, 1990; Leung,

[3] A closed-form solution of MI premium provides several advantages. First, one can better understand how sensitive MI premium is to the changes in relevant variables by conducting numerical analyses. Second, a closed-form solution can significantly increase the speed of calculation when the valuation of the MI premium is involved in more complicated analyses. Third, a closed-form solution of the MI premium provides a basic building block for MI insurers to design more complicated MI contracts. Because of previous reasons, we intend to support a closed-form pricing model for MI contract. The closed-form formula is important for MI insurers not only because it can be readily implemented in various market settings, but also because comparative statistics can be obtained analytically instead of by complicated numerical procedures. However, it is exceedingly difficult to derive a closed-form solution for MI valuation using an option-based model that includes more than two stochastic state variables (Kau et al., 1992, 1995; Kau, Keenan, and Muller, 1993; Kau and Keenan, 1995, 1999) because calculating the price requires solving a second-order partial differential equation that is subject to boundary and termination conditions. Thus, in general, only one stochastic state variable is used in option-based models.

2004; Girouard et al., 2006; Goodhart and Hofmann, 2008; Chang et al., 2012). These studies provide empirical evidence about the importance of macroeconomic factors in explaining the variations in housing prices. It implies that systematic risk driven by macroeconomic factors matters in interpreting the evolution of housing prices.

Idiosyncratic risk also plays an important role in housing returns, because the housing assets have restrictions on diversification due to high transaction costs and liquidity risk. Many studies find a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns (Eiling, 2006, Spiegel and Wang, 2006, Brockman and Schutte, 2007, and Fu, 2009). Capozza and Schwann (1990) find that nonsystematic risk can be a very important determinant of housing prices. Sanders (2008) emphasizes that the effects of regional economic factors are even more important than systematic risk effects in explaining U.S. mortgage defaults before 2005, although the systematic risk effect became more important from 2005 to 2008 Q2.

Moreover, several studies indicate that systematic risk (idiosyncratic risk) has a negative (positive) effect on housing returns (Capozza and Schwann, 1990; Miller and Pandher, 2008; Fei, 2009; Lee et al., 2014; Pu et al., 2016). According to the findings in the previous literature, one can infer that the systematic risk is indeed a crucial factor in determining mortgage insurance premiums. Furthermore, because MI insurers do not fully diversify in the reinsurance market, idiosyncratic risk should also be considered in MI valuation models.

In many MI models, the housing price change is assumed to follow a geometric Brownian motion without separate consideration of the systematic risk and the idiosyncratic risk (Kau et al., 1992, 1995; Kau, Keenan, and Muller, 1993; Kau and

Keenan, 1995, 1999; Bardhan et al., 2006). Pu et al. (2016) considered systematic and idiosyncratic shocks to price MI premiums. They use the correlation coefficient to decompose the volatility of the underlying housing prices into systemic volatility and idiosyncratic volatility. Such a decomposition method is usually used to address the problem of default correlation or default clustering in the pricing of financial products (Miao and Wang, 2007; Fan et al., 2012; Lee et al., 2015).

The important goal of this study was to model the housing price process including the systematic and idiosyncratic risk, thus can separate consider these two risks for pricing MI premiums. Such specification can work well for changes in housing prices caused by systematic risk or idiosyncratic risk. Several studies have demonstrated that the capital asset pricing model (CAPM) can effectively decompose the systematic risk and idiosyncratic risk on the housing return (Miller and Pandher, 2008; Sarama, 2010; Imreorow and Schagerstrom, 2011; Sarama, 2011; Voicu and Seiler, 2013). Thus, we adopt CAPM to model the housing price process including the systematic and idiosyncratic risk. Since our model focuses on investigating the influence of the systematic risk, driven by the entire financial market or the macroeconomic situation, on MI premiums,[4] our closed-form valuation formula can be readily implemented in various market settings.

For addressing the management of target PCR, we use the theory of VaR (value at risk) to derive a closed-form formula for an MI premium if the MI insurer seeks to restrain the systematic risk at a given confidence level. Using our pricing formula, MI

[4]The terms "systematic" and "systemic" risk have different definitions. According to the CAPM, systematic risks represent macroeconomic or aggregate risks that cannot be avoided through diversification. Systemic risk can be defined as the cascading or contagion effect of an idiosyncratic shock on the entire market due to the extensive interdependencies among firms in the market. Thus, this risk is sometimes considered to be different than systematic risk (Harrington, 2009). In addition, the clustering of mortgage defaults can be identified from the relevant literature to include systemic risk impacts (e.g., Harrington, 2009; Baluch et al., 2011). With regard to the insurance sector, systematic risk exposure can also be treated as a proxy for systemic risk (Allen and Jagtiani, 2000).

insurers can determine fair MI premiums according to changes in the economic environment. Thus, they can obtain MI premiums that adequately guard against losses caused by the systematic risk. Based on our model, it may also lower their extra reserve fund for satisfying the solvency capital requirements. That can help them achieve greater flexibility in the management of their liquidity.

A numerical example is provided to show how the model can be used in applications. We adopt the 51 U.S. state HPIs (housing price indexes), obtained from the Federal Housing Finance Agency (FHFA), as our source for the area-specific housing returns, and we use the S&P 500 Index and the U.S. HPI as the proxies of market portfolio. Then, we calculate the MI premium for each state. Through empirical analyses, we can understand which market portfolio proxy is better able to explain variation in the area-specific housing returns and thus is suitable for pricing MI.

This article contributes to the literature on MI contract pricing in the following ways. First, to the best of our knowledge, our pricing model is the first to provide a closed-form formula for MI valuation that considers both the systematic and idiosyncratic risks of the housing market. Such specification can fully depict the risks inherent in housing markets driven by common aggregate factors and idiosyncratic fluctuations arising from local shocks. Second, our model can be used to calculate MI premiums considering the systematic risk at different confidence levels. The model should help insurers determine reasonable MI premiums given their worst losses in a changing economic environment. Finally, we use U.S. housing market data to illustrate the application of our model and compare results from our model and those from a traditional model. Also, our numerical analyses demonstrate the sensitivity of MI premiums to changes in the model's parameters related to the systematic and idiosyncratic risks. This discussion should help insurers better understand how the different risks influence MI premiums.

The remainder of this paper is organized as follows. Section 2 provides the framework used for pricing MI premiums. It describes the components of the MI contract and defines the MI payouts that are made by the loss-given-default of the insurer. We first present the MI valuation model calculated by the traditional method, which does not consider the systematic and idiosyncratic risks. Then, we provide our MI valuation model, which includes these two risks, and derive the closed-form formula for MI considering the systematic risk at a given confidence level. In Section 3, we provide a numerical example to demonstrate the application of our model. This section also includes a discussion of the sensitivity of MI premiums to changes in the model's parameters and compares results from our model and the traditional model. In the final section we summarize our findings.

2. The models

This section presents the valuation framework for an MI premium. In Subsection 2.1, we describe the components of MI and introduce the traditional MI pricing model provided by Bardhan et al. (2006). Subsection 2.2 explains how to derive the closed-form formula for the MI premium when the valuation model considers the systematic and idiosyncratic risks.

2.1 The traditional MI valuation model

Numerous studies have used options-based pricing models for pricing and analyzing MI contracts. However, it is exceedingly difficult to derive a closed-form formula for MI valuation using an option-based model that includes more than two stochastic state variables. Bardhan et al. (2006) is the first paper to present an option-pricing framework for pricing MI contracts in a closed-form. In this literature, it includes only one stochastic state variable (i.e., housing prices) and assume the default probability to be an exogenous variable.[5]

Our study uses the fixed-rate mortgage (FRM), which is the basic building block of the mortgage market. Let the FRM be a fully amortized mortgage with a fixed coupon rate c and time to maturity of T years. At origination, the initial mortgage principal is obtained as follows:

$$M(0) = \eta H(0)\,, \tag{1}$$

where η is the LTV and $H(0)$ is the initial house price. This implies a paymant Y equal to:

$$Y = cM(0)\,(1 - \frac{1}{(1+c)^T})^{-1}\,. \tag{2}$$

The principal outstanding at time t, $M(t)$, is obtained by

$$M(t) = \frac{Y}{c}(1 - \frac{1}{(1+c)^{T-t}})\,. \tag{3}$$

Generally, there are two basic kinds of MI available in the U.S. financial market: private mortgage (PM) insurance and government mortgage insurance, offered mainly by the Federal Housing Administration (FHA). MI guarantees that if a borrower defaults on a loan, a mortgage insurer will pay the mortgage lender for any loss resulting from a property foreclosure, up to a certain percentage of the claim amount. They differ along such dimensions as the depth of coverage offered. The FHA is obligated to pay all the losses, whereas PM insurers are obligated to cover the

[5] Bardhan et al. (2006) mention that one way of estimating these probabilities is by using actuarial mortgage default and prepayment experience, or their proxies if the appropriate data are unavailable. Moreover, as long as past prepayment and default experience are decent predictors of future prepayment and default experience (a reasonable assumption in stable economies), such an approach is guaranteed to work; i.e., on average, the modeled unconditional probabilities of default coincide with the observed ones.

maximum loss, which usually ranges between 20% and 30% of the exposure.[6] This means that in the event of default, the insurer pays the difference between the house's value and the mortgage payment, up to some given fraction of the unpaid balance. In this study, we support the valuation model for the MI of FHA.

According to the typical model of an MI contract (Kau et al., 1995, Bardhan, et al., 2006; Chen et al., 2009), the realized loss for the insurer in case the borrower defaults can be represented as a portfolio of put options on the borrower's collateral. Following their model, if a default occurs at time τ, the insurer has to pay the lender the amount described as follows:

$$L(\tau) = \max(0, \min(M(\tau-1) - H(\tau), l_R M(\tau-1)), \tag{4}$$

where $L(\tau)$ is the payoff of the MI insurer at time τ and l_R is the loss ratio. This equation implies that if the collateral value is greater than the outstanding loan, the lender can fully recover the outstanding loan from the foreclosure or short sale proceeds, and thus the loss to the insurer is zero. On the other hand, if the housing price is not sufficient for a full repayment of the loan balance, the maximum loss the insurer is obligated to pay is equal to $l_R M(\tau-1)$.

Equation (4) can be rewritten as follows:

$$L(\tau) = \max(0, M(\tau-1) - H(\tau)) - \max(0, (1-l_R)M(\tau-1) - H(\tau)). \tag{5}$$

The present value of the severity of loss, $PL(\tau)$, is given by the following expression:

$$\begin{aligned} PL(\tau) &= E[e^{-r\tau} L(\tau)] \\ &= E[e^{-r\tau} \max(0, K_1 - H(\tau))] - E[e^{-r\tau} \max(0, K_2 - H(\tau))], \end{aligned} \tag{6}$$

where $E[\cdot]$ is an expected operator under the risk-neutral measure, $K_1 = M(\tau-1)$, and $K_2 = (1-l_R)M(\tau-1)$. Equation (6) shows that the expected present value of

[6] For more detailed information, please see the website http://www.alliemae.org/pmi.html.

$L(\tau)$ equals the difference between two European put options with the same underlying asset $H(\tau)$ and the time to maturity equaling the time to default τ. Thus, the expected present value of $L(\tau)$ can be duplicated by a long position in a European put option with a strike price K_1 and a short position in a European put option with a strike price K_2; that is, its payoff is the same as the payoff from a bear spread created by the two put options. For simplicity, we represent Equation (6) as the option style that includes two European put options with different strike prices as follows:

$$PL(\tau) = P(K_1, \tau) - P(K_2, \tau), \tag{7}$$

where $P(K_i, \tau)$ is a European put option with underlying asset $H(\tau)$, strike price K_i, $i = 1,2$ and maturity date τ.

To solve the closed-form formula for these two European put options in Equation (6), the evolution of the house return needs to be specified. In the previous relevant studies on MI pricing, the housing price change is assumed to follow a traditional geometric Brownian motion (Kau et al., 1992, 1995; Kau, Keenan, and Muller, 1993; Kau and Keenan, 1995, 1999; Bardhan et al., 2006). It can be expressed as follows:

$$\frac{dH(t)}{H(t)} = (r - \delta)dt + \sigma_H dW_H(t), \tag{8}$$

where $H(t)$ is the housing price, r is the risk-free interest rate, δ is the depreciated yield, σ_H is the instantaneous standard deviation of the housing return and $W_H(t)$ is a standard Brownian motion of the housing return under the risk-neutral measure.

Using the standard pricing results for European put options with constant dividend yields (Hull, 1999), the two European put options in Equation (7) can be solved. Their closed-form formulas can be expressed as

$$P(K_i,\tau) = K_i e^{-r\tau} N(-d_2(K_i)) - H(0)e^{-\delta\tau} N(-d_1(K_i)), \tag{9}$$

where $N(\cdot)$ is the cumulative density of a standard random variable;

$$d_1(K_i) = \frac{\ln(\frac{H(0)}{K_i}) + ((r-\delta) + \frac{1}{2}\sigma_H^2)\tau}{\sigma_H \sqrt{\tau}}$$; and

$$d_2(K_i) = d_2(K_i) - \sigma_H \sqrt{\tau} .$$

The actuarially fair price has an expected net payoff of zero; that is, the premiums paid are equal to the expected value of the compensation received. Such an insurance policy makes no economic profit. Thus, the actuarially fair price of the MI premium is determined as the sum of the expected loss for each time point of the life of the mortgage. Since, by assumption, the housing price is independent of the unconditional probability of the borrower's default $P_d(\tau)$, the fair price of the MI premium can be expressed as follows:

$$MI^T = \sum_{\tau=1}^{T} P_d(\tau) PL(\tau), \tag{10}$$

where MI^T is the MI premium calculated by the Bardhan et al. (2006) model, and $P_d(\tau)$ is the unconditional default probability at time τ. The next subsection illustrates how the systematic and idiosyncratic risks are incorporated in the valuation model for the MI premium.

2.2 The MI valuation model considering systematic and idiosyncratic risks

This study refers to the model shown in Bardhan et al. (2006) as the basic model for valuing an MI premium. Unlike Bardhan et al. (2006), our model emphasizes the importance of both systematic and idiosyncratic risks in driving the evolution of housing returns, and thus the MI premiums should be more accurately measured by our model than theirs.

A number of studies have demonstrated that the CAPM, which uses the return of the aggregate U.S. housing market as the market portfolio proxy, can effectively decompose the systematic risk and idiosyncratic risk on the housing return (Capozza and Schwann, 1990; Miller and Pandher, 2008; Sarama, 2010; Imreorow and Schagerstrom, 2011; Case et al., 2011; Sarama, 2011; Voicu and Seiler, 2013). Some studies also computed the average idiosyncratic volatility based on the variance decomposition or used the residuals from the CAPM (Capozza and Schwann, 1990; Miller and Pandher, 2008). We therefore use the CAPM to decompose the risk of underlying housing prices into the systematic risk and the idiosyncratic risk.

Using the CAPM in the investigation related to the housing market, we have

$$r_H = \kappa + \beta r_M + \varepsilon , \tag{11}$$

where r_H is the housing return in a regional housing market; κ is the intercept term; β is the coefficient, which is a measure of the systematic risk of a regional housing market in comparison to the whole market; r_M is the return of the market portfolio; and ε is the error term for regression. According to Equation (11), the relationship between housing market volatility and whole market volatility can be expressed as follows:

$$\sigma_H^2 = \beta^2 \sigma_M^2 + \sigma_\varepsilon^2 . \tag{12}$$

In this equation, $\beta^2 \sigma_M^2$ denotes the systematic risk, where σ_M^2 is the variance of the return of the market portfolio. Since we assume that the market portfolio proxy can capture all the systematic risk, by definition only the idiosyncratic risk is left in the residuals. Thus, we denote σ_ε^2 as the idiosyncratic risk.

In order to properly model the housing price process including these two risks, we decompose the volatility of the underlying housing prices into systematic volatility and idiosyncratic volatility. Thus, $\sigma_H dW_H(t)$ in Equation (8) becomes as follows:

$$\sigma_H dW_H(t) = \beta\sigma_M \, dW_M(t) + \sigma_\varepsilon dW_\varepsilon(t), \tag{13}$$

where $W_M(t)$ and $W_\varepsilon(t)$ are the standard Brownian motion. Here we assume $W_M(t)$ and $W_\varepsilon(t)$ are independent. Then, according to Equation (12), we have

$$\sigma_H^2 dt = \beta^2\sigma_M^2 dt + \sigma_\varepsilon^2 dt. \tag{14}$$

Next, we derive the closed-form formula for evaluating the actuarially fair price of an MI premium based on the probability of a loss rate caused by the systematic risk that is not exceeded by 1-α confidence level in VaR theory. According to Equation (6), the value of a European put option based on the systematic risk at the 1-α confidence level (denoted as $P^\alpha(K_i,\tau)$) can be represented as follows:[7]

$$P^\alpha(K_i,\tau) = K_i E[e^{-r\tau} I_{D^\alpha(\tau)}] - E[e^{-r\tau} H(\tau) I_{D^\alpha(\tau)}], \tag{15}$$

where $D^\alpha(\tau)$ is a set of $H(\tau) < K_i$ with the systematic risk at the 1-α confidence level, and $I_{D^\alpha(T)}$ is an indicator operator equal to 1 if $D^\alpha(\tau)$ occurs and 0 otherwise.

On the right-hand side of Equation (15), $K_i E[e^{-r\tau} I_{D^\alpha(\tau)}] = K_i e^{-r\tau} E[I_{D^\alpha(\tau)}]$. When deriving the closed-form formula for MI premiums, we need to solve $E[I_{D^\alpha(\tau)}]$

[7] For simplicity, the interest rate in our model is not assumed to be a random variable because previous research does not identify any effect of interest rate volatility on mortgage insurance costs (Hendershott and Van Order, 1987).

and $E[e^{-r\tau}H(\tau)I_{D^{\alpha}(\tau)}]$, where $E[I_{D^{\alpha}(\tau)}]$ is the probability of the set $(H(\tau)<K_i)$ considering the systematic risk given 1-α confidence level. To obtain these two expected values, we need to specify the distribution for $dW_H(t)$. Since $dW_l(t)$, $l=M,H,\varepsilon$, is normally distributed, we denote it as $N(0,dt)$; and we define a random variable Z_l, $Z_l=\frac{\int_0^{\tau}dW_l(s)}{\sqrt{\tau}}=\frac{W_l(\tau)}{\sqrt{\tau}}$, which follows a standardized normal distribution. Thus, we obtain

$$\sigma_H dW_H(\tau)\equiv\sigma_H\sqrt{\tau}Z_H; \tag{16}$$

$$\sigma_M dW_M(\tau)\equiv\sigma_M\sqrt{\tau}Z_M;\text{ and} \tag{17}$$

$$\sigma_\varepsilon dW_\varepsilon(\tau)\equiv\sigma_\varepsilon\sqrt{\tau}Z_\varepsilon. \tag{18}$$

Moreover, according to Equation (7), we have

$$H(\tau)=H(0)\exp((r-\delta-\frac{1}{2}\sigma_H^2)\tau+\int_0^{\tau}\sigma_H dW_H). \tag{19}$$

When the housing return process includes both systematic and idiosyncratic risks, the dynamic process of the housing price becomes

$$H(\tau)=H(0)\exp((r-\delta-\frac{1}{2}(\beta^2\sigma_M^2+\sigma_\varepsilon^2))\tau+\int_0^{\tau}\beta\sigma_M dW_M+\int_0^{\tau}\sigma_\varepsilon dW_\varepsilon). \tag{20}$$

Accordingly, the probability of $H(\tau)<K_i$ at time τ given a deterministic value of Z_M can be expressed as follows:

$$P(z_M)\equiv\Pr(H(\tau)<K_i|Z_M=z_M)$$

$$=N(\frac{\ln(\frac{K_i}{H(0)})-(r-\delta-\frac{1}{2}(\beta^2\sigma_M^2+\sigma_\varepsilon^2))\tau-\beta\,\sigma_M\sqrt{\tau}z_M}{\sigma_\varepsilon\sqrt{\tau}}), \tag{21}$$

where $P(z_M)$ is the probability that the house price is less than K_i given a specified value of Z_M at time τ, and $\Pr(\cdot)$ is an operator of probability.

We use the method shown in Lee et al. (2015) to obtain the MI premium considering a systematic risk based on VaR theory. We let X be a random variable which denotes the probability of $H(\tau) < K_i$ given z_M, i.e., $X \equiv P(z_M)$. We assume that the MI company intends to let the default probability be less than a deterministic value x (i.e., $X \le x$). Thus, the cumulative probability of X being less than x can be expressed as follows (see Lee et al. , 2015):

$$F(x) = \Pr(X \le x) = \int_{-z_M^*}^{+\infty} n(x)dx = N(z_M^*), \tag{22}$$

where $\Pr(\cdot)$ is the probability operator, $n(\cdot)$ is the probability density function of the standard normal distribution, and z_M^* is the critical value of $\Pr(X \le x)$.

Next, we explain how MI insurers can calculate the MI premium under the systematic risk, given the 1-α confidence level. Because β is used to describe the relationship between the return of an asset or a portfolio, and the return of the market portfolio, its value can be negative or positive. According to Equation (11), if $\beta > 0$, a decrease in the market portfolio return decreases the return of the regional housing market and then increases the value of X (i.e., the probability of $H(\tau) < K_i$). Thus, if we want to let the default probability be less than a certain value (i.e., $\Pr(X \le x)$), we should stress the maximum possible decrease in the market portfolio return. For this reason, we should focus on the left-hand tail of the distribution of market portfolio returns. In contrast, if $\beta < 0$, a decrease in the market portfolio return increases the return of the regional housing market, and then decreases the value of X. Thus, if we intend to let the default probability be less than a certain value, we should attach importance to the maximum possible increase in the market portfolio return. In this case, the analyses should concentrate on the right-hand tail of the distribution of market portfolio returns.

Then, we show how to solve $E[I_{D^{\alpha}(\tau)}]$ under $\beta > 0$ and $\beta < 0$. To begin with, in the case of $\beta > 0$, we need to know the range of z_M given $P(z_M) \le x$. Based on Equation (19), we have

$$\frac{\ln(\frac{K_i}{H(0)}) - (r - \delta - \frac{1}{2}(\beta^2\sigma_M^2 + \sigma_\varepsilon^2))\tau - \beta\,\sigma_M\sqrt{\tau}z_M}{\sigma_\varepsilon\sqrt{\tau}} \le N^{-1}(x). \tag{23}$$

From the above inequality, we can derive the range of z_M given $P(z_M) \le x$, as shown below:

$$\begin{aligned} z_M &\ge \frac{\sigma_\varepsilon\sqrt{\tau}}{\beta\,\sigma_M\sqrt{\tau}}\left(\frac{\ln(\frac{K_i}{H(0)}) - (r - \delta - \frac{1}{2}(\beta^2\sigma_M^2 + \sigma_\varepsilon^2))\tau}{\sigma_\varepsilon\sqrt{\tau}} - N^{-1}(x)\right) \\ &= \frac{\sigma_\varepsilon}{\beta\,\sigma_M}(-d_2^S(K_i) + \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} - N^{-1}(x)), \end{aligned} \tag{24}$$

where $d_2^S(K_i) = \dfrac{\ln(\frac{H(0)}{K_i}) + (r - \delta - \frac{1}{2}\sigma_\varepsilon^2)\tau}{\sigma_\varepsilon\sqrt{\tau}}$. According to Equations (22) and (24), z_M^*, which is the critical value of $\Pr(X \le x)$, can be obtained as follows:

$$z_M^* = \frac{\sigma_\varepsilon}{\beta\,\sigma_M}(d_2^S(K_i) - \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + N^{-1}(x)). \tag{25}$$

Afterward, we illustrate how to derive the critical value of x at the $1-\alpha$ confidence level (denoted as $x_{1-\alpha}$). This value, which represents the probability of a loss caused by a systematic risk equal to $1-\alpha$, can be expressed as follows:

$$1-\alpha = F(x_{1-\alpha}) = N(\frac{\sigma_\varepsilon}{\beta\,\sigma_M}(d_2^S(K_i) - \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + N^{-1}(x_{1-\alpha}))), \tag{26}$$

Then, we can obtain

$$N^{-1}(1-\alpha) = \frac{\sigma_\varepsilon}{\beta\,\sigma_M}(d_2^S(K_i) - \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + N^{-1}(x_{1-\alpha})). \tag{27}$$

and the value of $x_{1-\alpha}$ becomes

$$x_{1-\alpha} = N(-d_2^S(K_i) + \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + \frac{\beta\,\sigma_M}{\sigma_\varepsilon}N^{-1}(1-\alpha)).$$

In Equation (26), $x_{1-\alpha}$ is the probability of $V(T) < B(T)$ given the systematic risk at the $1-\alpha$ confidence level. In other words, we can obtain the value of $E[I_{D^\alpha(\tau)}]$ as

$$E[I_{D^\alpha(\tau)}] = x_{1-\alpha} = N(-d_2^S(K_i) + \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + \frac{\beta\,\sigma_M}{\sigma_\varepsilon}N^{-1}(1-\alpha)). \qquad (28)$$

Moreover, in light of the above equation, for $\beta < 0$, we have

$$E[I_{D^\alpha(\tau)}] = x_{1-\alpha} = N(-d_2^S(K_i) + \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} - \frac{\beta\,\sigma_M}{\sigma_\varepsilon}N^{-1}(1-\alpha)). \qquad (29)$$

For simplicity, we represent $E[I_{D^\alpha(\tau)}]$ as follows:

$$E[I_{D^\alpha(\tau)}] = N(-d_2^S(K_i) + \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} + \frac{|\beta|\sigma_M}{\sigma_\varepsilon}N^{-1}(1-\alpha)), \qquad (30)$$

where $|\beta|$ is the absolute value of β.

Next, we solve the second term on the right-hand side of Equation (15), $E[e^{-r\tau}H(\tau)I_{D^\alpha(\tau)}]$. Let $\xi(\tau) = \exp((-\frac{1}{2}\sigma_H^2)\tau + \int_0^\tau \sigma_H dW_H)$ be a Radon-Nikodym derivative. After changing the measure and following the above procedures, we have

$$E[e^{-r\tau}H(\tau)I_{D^\alpha(\tau)}] = H(0)e^{-\delta\tau}N(-d_1^S(K_i) - \frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_\varepsilon} - \frac{|\beta|\sigma_M}{\sigma_\varepsilon}N^{-1}(1-\alpha)), \quad (31)$$

where $d_1^S(K_i) = \dfrac{\ln(\frac{H(0)}{K_i}) + ((r-\delta) + \frac{1}{2}\sigma_\varepsilon^2)\tau}{\sigma_\varepsilon\sqrt{\tau}}$.

Then, putting the above solutions $E[I_{D^{\alpha}(\tau)}]$ (i.e., Equation (30)) and $E[e^{-r\tau}H(\tau)I_{D^{\alpha}(\tau)}]$ (i.e., Equation (31)) into Equation (15), the value of the European put option based on the systematic risk at the 1 - α confidence level can be obtained as follows:

$$P_{\alpha}(K_i,\tau)=K_i e^{-r\tau}\ N(-d_2^S(K_i)+\frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_{\varepsilon}}+\frac{|\beta|\sigma_M}{\sigma_{\varepsilon}}N^{-1}(1-\alpha))$$

$$-H(0)e^{-\delta\tau}N(-d_1^S(K_i)-\frac{\beta^2\sigma_M^2\sqrt{\tau}}{2\sigma_{\varepsilon}}-\frac{|\beta|\sigma_M}{\sigma_{\varepsilon}}N^{-1}(1-\alpha)). \tag{32}$$

Accordingly, the present value of the severity of the loss for the MI insurer based on the systematic risk at the 1 - α confidence level can be expressed as follows:

$$PL^{\alpha}(\tau)=P^{\alpha}(K_1,\tau)-P^{\alpha}(K_2,\tau). \tag{33}$$

Finally, we follow the assumption shown in Bardhan et al. (2006) that the unconditional probability of borrower default, $P_d(\tau)$, is an exogenous variable.[8] The actuarially fair price of the MI premium MI^S is as follows:

$$MI^S=\sum_{\tau=1}^{T}P_d(\tau)PL^{\alpha}(\tau). \tag{34}$$

Equation (34) is a closed-form formula for MI premiums that considers both systematic and idiosyncratic risks. It can be used to calculate MI premiums under various economic conditions over time at the 1-α confidence level. The model provides reasonable and accurate values for the MI premium because it simultaneously considers the effects of systematic and idiosyncratic risks on MI premiums. In the next section, we show that the MI premium is underestimated if it has not considered the systematic risk and idiosyncratic risk.

3. Numerical Analyses

[8] Such specification has been found in many studies related with the valuation of MI premium. For example, Schwartz and Torous (1993), Dennis, Kuo and Yang (1997), and Bardhan et al. (2006), Chen et al. (2010) among others, model the unconditional probability of default exogenously.

Here we use numerical examples to illustrate the application of our model. One can apply our model to deal with the deal with different kinds of MI premiums, such as the individual-level MI, the city-level MI, the area-level MI, or the state-level MI. For example, if the MI companies want to decide the MI for a particular area, they can apply our model by using by the HPI for this area. Previous studies emphasizes that the effects of regional economic factors are even more important than effects of systematic risk in explaining U.S mortgage defaults before 2005 (Sanders, 2008). In the empirical section of this study, we evaluate the state-level MI premium. Thus, we use the state-level HPI relative to the country-level HPI to be the example for illustrating the application of our model.

Several studies related to the housing return empirically demonstrate that the CAPM, which uses the S&P 500 Index as a factor of market risk, can explain differences in systematic risk across cities (Jud and Winkler, 2002; and Anderson and Beracha, 2010). Jud and Winkler (2002) show that lagged stock market returns have a significant positive impact on MSA house price returns. Anderson and Beracha (2010) find a positive relationship between the returns of houses in more than 3000 U.S. zip-codes and the stock market. Nevertheless, a number of authors argue that the U.S. real estate index has much more explanatory power than the S&P 500 Index (Chinloy, 1992; Davidoff, 2007; Case et al., 2011; Voicu and Seiler, 2013). Several studies related to investigations of the housing market show that the market-wide housing return is a significant systematic risk-factor in explaining the area-specific real estate returns (Cannon et al., 2006; Case et al., 2011). Thus, we respectively adopt the S&P 500 Index and the return of the aggregate U.S. housing market index, that is, the Housing Price Index (HPI) obtained from the Federal Housing Finance Agency (FHFA), as the market portfolio proxy.

As for the trends of international developments, the solvency regimes differ among countries.[9] The aim of a solvency regime is to ensure the financial soundness of insurance undertakings, and in particular to ensure that they can survive difficult periods. Solvency rules stipulate the minimum amounts of financial resources that insurers and reinsurers must have in order to cover the risks to which they are exposed. Equally important, the rules also lay down the principles that should guide insurers' overall risk management so that they can better anticipate any adverse events and better handle such situations. Since the idiosyncratic risk in our model derives from the regional housing market, we adopt the 51 U.S. state HPIs as the housing returns for the regions in the housing market.

The S&P 500 Stock Index data were taken from the TEJ databank.[10] In this study, all data on the housing price indices (i.e., aggregate national HPI and the HPI of the 51 states in the U.S.) were obtained from the Federal Housing Finance Agency (FHFA).[11] The FHFA's HPI is thoroughly documented in the real estate literature (Fei, 2009; Case et al., 2011; Voicu and Seiler, 2013).[12] The one reason is that these indices use repeat sales of all properties with mortgages purchased or securitized by Fannie Mae and Freddie Mac since 1975. The repeat sales methodology, based on work by Case and Shiller (1987), is attractive because it keeps quality constant across time. Another reason the FHFA indices are attractive is that they provide the broadest coverage of U.S. metropolitan areas (381 cities) and a longer time series than other price index sources.

[9] The main representatives of the world's regulatory systems are the solvency regimes of the EU (Solvency I、Solvency II) and the NAIC's RBC (risk-based capital) regime of the U.S.

[10] TEJ is a well-known databank containing financial information on Taiwan.

[11] The website is http://www.fhfa.gov/.

[12] The OFHEO and the Federal Housing Finance Board were combined in July 2008 through the Housing and Economic Recovery Act of 2008 to emerge as the Federal Housing Finance Agency. The FHFA's HPI index is the same as the OFHEO's HPI index cited in the previous literature.

Here we adopt data from the U.S. HPI's seasonally adjusted purchase-only index. The seasonally adjusted HPI for each state is used as the area-specific housing market index. The U.S. HPI and each state's HPI are quarterly data collected from 1991Q1 to 2017Q2, yielding 105 samples in each state. Table 1 presents the means, standard deviations, and the maximum, median, and minimum values for each variable, including the S&P 500 Index, the U.S. HPI, and all the state HPIs.

[Insert Table 1 here]

Next, we illustrate how to apply our model. We selected a fixed-rate mortgage contract with 30-year maturity for our example. To illustrate the implementation of our model, we first assign the following base parameters: $T = 30$ years, $c =$ 3% (mortgage rate), r = 2% (interest rate), η = 95% (loan-to value ratio), $H(0)$ = \$100 thousand (initial house value), $M(0)$ = 95 thousand (initial value of mortgage), δ = 1 % (house depreciation rate), and l_R = 75% (loss ratio). Data pertaining to the default probabilities were obtained from the Department of Housing & Urban Development's 2010 FHA annual actuarial report.[13] These data were sampled yearly from 1998 to 2010. Table 2 shows the means of reported default probabilities for each calendar year.

[Insert Table 2 here]

We use Equations (11) and (12) to estimate beta (β) for the CAPM, the systematic risk, and the idiosyncratic risk. We multiply the series of quarterly returns of the market portfolio and each state's HPI by 4 to obtain the series of yearly returns. In Table 3, we show the estimates of only one state (Alaska) as an example. In this

[13] The title of the report is "Actuarial Review of the Federal Housing Administration Mutual Mortgage Insurance Fund (Excluding HECMs) for Fiscal Year 2010." HUD's website is http://portal.hud.gov/hudportal/HUD.

case, the standard deviation of the housing return is 0.0521 ($\sigma_H =$ 0.0521). If we use the U.S. HPI as the proxy for market portfolio, $\beta = 0.4260$ (significant at the 1% level); the standard deviations for the return of the U.S. HPI is $\sigma_M = 0.0482$ and the standard deviations for residual is $\sigma_\varepsilon =$ 0.0479. If we use the S&P 500 as the proxy for market portfolio, we have $\beta = -0.0261$; the standard deviations for the returns of the S&P 500 Stock Index are $\sigma_M = 0.3013$ and the standard deviations for residual is $\sigma_\varepsilon = 0.0515$. However, the estimate of β is not significant when using the S&P 500 Stock Index as the proxy for market portfolio. In this case, the U.S. HPI would have been a more suitable proxy for the market portfolio.

[Insert Table 3 here]

Using the same method, we estimate β, the systematic risk, and the idiosyncratic risk for the 51 U.S. states. Tables 4 and 5 summarize the estimates of β for each U.S. state,[14] respectively adopting the U.S. HPI and S&P 500 for the market portfolio. As shown in Table 4, all the β estimates are significant at the 1% level using the U.S. HPI as the proxy for the market portfolio. The β estimates are all positive, ranging from 0.2106 for North Dakota (ND) to 2.5173 for Nevada (NV). In other words, there is a positive relationship between the U.S. HPI returns and the state HPI returns for all states. However, in Table 5, the β estimates are not all significant or positive; they are significant for only four states and none are significant at the 1% level. They range from -0.0541 for Hawaii (HI) to 0.0709 for Nevada (NV).

[Insert Tables 4 and 5 here]

These results are consistent with previous studies, whose authors argue the U.S. HPI has much more explanatory power than the S&P 500 Index (Chinloy, 1992;

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[14] Appendix A gives the code names of the 51 U.S. states (including the District of Columbia).

Davidoff, 2007; Case et al., 2011; Voicu and Seiler, 2013). The market-wide housing return is a significant systematic risk-factor in explaining the area-specific real estate returns (Cannon et al., 2006; Case et al., 2011). Furthermore, empirical results of several studies show that the estimated β values range from -0.185 to 2.6 (Chinloy, 1992; Case, Cotter and Gabriel. 2011; Voicu and Seiler, 2013). Thus, our estimates of βs ranging from 0.2106 to 2.5173 are reasonable since their values are within the scope of the estimated βs in previous studies. Because our results reveal that the U.S. HPI should be a more suitable proxy for the market portfolio, our follow-up analyses focus only on the cases with the U.S. HPI as this proxy.

In Table 6 we show the ratios of the idiosyncratic risk to the whole housing risk for each state. When using the U.S. HPI return for the market portfolio, these risks range from 32.72% to 97.64%. These ratios are larger than 50% for 47 of the 51 states. Thus, our results reveal that compared with the systematic risk, the idiosyncratic risk indeed plays an important role in the whole housing risk, because the housing assets have restrictions on diversification. This result is similar to the findings of Capozza and Schwann (1990), where the unsystematic risk is a larger proportion of the total risk than is the systematic risk. Therefore, one can infer that most of the effect of the total housing risk can be ascribed to the idiosyncratic risk.

[Insert Table 6 here]

After obtaining the estimates for β and σ_ε from Equation (11), we calculate the value of $\beta\,\sigma_M$ and further calculate the MI premium based on our formula. We give the systematic risk at the 95% confidence level, i.e., α = 5%. Next, we compare the results calculated from our model and from the traditional model provided by Bardhan et al. (2006). Here the estimated MI premium is expressed as the percentage of the initial loan value. Table 7 summarizes the estimates of the MI premium for each state using the U.S. HPI for the market portfolios. In Table 7, we use MI^T and MI^S_{HPI} to represent the MI premiums calculated by the traditional method and our method with the U.S. HPI index as the proxy for market portfolio. As shown in this table, the MI premiums using the traditional method are all quite low. For example, in Alaska we have MI^T = 0.0048%. In our model, we have MI^S_{HPI} =0.2767%. In Table 7, the results reveal that the MI premiums calculated by our model are all higher than those calculated by the traditional method.

[Insert Table 7 here]

Currently, the FHA's MIP has two components: the upfront premium (UFMIP) and the annual premium. The current upfront premium rate is 1.75% of the loan amount and the current annual premium is 0.85% for the most common category of FHA loans.[15] The FHA does not vary its premium based on borrower risk. Unlike the FHA's MI insurance, the values of PM's MI premiums depend on the borrower's credit risk. The lowest charge for the high-FICO-score borrowers (above 760) is 0.55% and the highest charge for the low-FICO-score borrowers (620-639) is 2.25%. The MI premiums for the FHA and PMI are the percentages of the mortgage's outstanding balance.

Table 8 shows the statistical summary for the estimated MI premiums. The average values of MI^T and MI^S_{HPI} are 0.0364% and 1.0562%, respectively. The average value of MI^S_{HPI} is larger than the average value of MI^T. The minimum and maximum values of MI^T are 0.0001% and 0.3472%. The minimum and maximum values of MI^S_{HPI} are 0.0207% and 5.2733%. The MI premiums calculated by our model, considering both the systematic and idiosyncratic risks, are higher than those

[15] The most common category of FHA loans refers to LTVs of 95% or above, loans of $625,000 or below and payments for the term of the mortgage.

calculated by the model without considering these two risks. Our model gives a more reasonable valuation of the MI premium because it more sensitively reflects the housing market risk. According to Table 8, the MI premiums from the traditional method are unreasonable because they are much lower than the current MI premiums no matter whether the source is the FHA or PM. The MI premiums calculated by our model seem also lower than the current MI premiums required by the FHA and PM. However, the values in our model are closer to the current required MI premiums than that in traditional model. The MI premiums of the FHA and PM consider the LTV, loan amount, payments for the term of the mortgage and the borrower's credit risk, but MI premiums of the FHA and PM do not consider both systematic and idiosyncratic risks. Thus, our model should help MI insures accurately determine fair MI premiums based on the different systematic and idiosyncratic risks and to effectively undertake sophisticated risk management.

[Insert Table 8 here]

To analyze the influence of the 2007 financial crisis on MI premiums, we also estimate MI premiums corresponding to the crisis. We divide the sample period into two sub-periods: Before Financial Crisis and After Financial Crisis. Before Financial Crisis is defined as the period from 1991Q2 to 2007Q1 and After Financial Crisis is defined as the period from 2007Q2 to 2017Q2. Table 9 gives the statistical summary. The means of the returns of the U.S. HPI (denoted as r_{HPI}) are 5.2604% and 0.9719% for the periods before and after the crisis, respectively, and the corresponding standard deviations are 2.7852% and 5.8659%. These results reveal that the housing price declined and the systematic risk increased because of the financial crisis.

[Insert Table 9 here]

Table 9 shows that the means (standard deviations) of MI^T are 0.0209% (0.0637%) and 0.0543% (0.1014%) for the periods before and after the crisis, respectively. The corresponding means (standard deviations) of MI^S_{HPI} are 0.6207% (1.0236%) and 1.3368% (1.5859%). The results reveal that regardless of whether the MI premiums are calculated by the traditional model or our model, the MI premium is shown to have increased due to the financial crisis. Accordingly, the MI premiums indeed went up because of the increase in the housing risk caused by the 2007 financial crisis.

Next, we describe our sensitivity analyses of the impact of the model's parameters related to the systematic and idiosyncratic risks (α, β, σ_M and σ_ε) on the MI premium. The relevant parameters are given the following values: $T = 30$ years, $c =$ 3%, $r =$ 2%, $\eta =$ 95%, $H(0) =$ 100 thousand, $M(0) =$ 95 thousand, δ =1%, l_R =75%, and we let α range from 0.01 to 0.5. Figure 1 shows the relationship between the MI premium (MI^S) and the confidence level of the systematic risk (α). This figure shows the MI premium to be a convex function with a negative slope with respect to α. In other words, as the value of α increases, the MI premium first sharply decreases and then slowly decreases.

[Insert Figure 1 here]

Figure 2 shows how the values of MI^S vary corresponding to changes in the value of β. Here we assume that β ranges from 0 to 2. This figure shows that the relationship between MI^S and β is a bit similar to an S-shaped curve with a positive slope. It tells us that MI^S is positively related to β; a rise in β causes first a steady increase, then a steep increase and finally a steady increase in the change of MI^S. Moreover, Figures 3 and 4 show how sensitive MI^S is to changes in σ_M and σ_ε. According to Figure 3, the influence of a change in σ_M on the value of MI^S is similar to the influence of a change in β on the value of MI^S. The relation of MI^S and σ_M is also a bit similar to S shape curve with a positive slope. Figure 4 tells us that MI^S is positively related to σ_ε; a rise in σ_ε first causes a sharp increase in the change of MI^S followed by a very slow increase. In view of this, when MI insurers undertake risk management, idiosyncratic risks must be considered. To sum up, from the findings of our numerical example we infer that the MI premium is positively related to the parameters of both the systematic and unsystematic risks.

[Insert Figures 2-4 here]

4. Conclusion

The sub-prime mortgage crisis that occurred in the U.S. in 2007 produced large losses from default for mortgage insurance institutions. Such systematic risk has posed new and considerable challenges on the pricing of mortgage insurance premiums. Moreover, since the housing assets have restrictions on diversification due to high transaction costs and liquidity risk, MI insurers do not fully diversify through the reinsurance market, thus incurring idiosyncratic risk. A large number of previous studies have demonstrated that systematic and idiosyncratic risks are indeed a crucial factor in the evolution of housing prices. For the above reasons, this study was undertaken to provide a reasonable model including these two risks to fairly valuate an MI premium.

When evaluating and analyzing MI contracts, numerous authors have adopted an options-based pricing model. Bardhan et al. (2006) were the first to develop a closed-form formula for MI contracts based on an option-pricing framework;

thereafter, a number of studies using options-based pricing models to price MI premiums followed the model provided by Bardhan et al. (2006). Because the closed-form formula can be readily implemented in various market settings and can avoid complicated numerical procedures, we valuate MI premiums based on the option-pricing framework provided by Bardhan et al. (2006). Further, we used market data to conduct a numerical analys, and then compared the results from our model and from the traditional model provided by Bardhan et al. (2006). Doing so allows one to understand the effect of systematic and idiosyncratic risks on MI premiums.

A number of studies have provided empirical evidence demonstrating that the CAPM provides a good explanation of the relationship between the housing return and the systematic risk. We therefore used the theory of CAPM to deal with the systematic and idiosyncratic risks. In addition, efficiently managing the exposure to the risks is important for MI insurers, because it can avoid the losses and may lower their capital requirements for holding insured mortgage loans. In considering the target PCR for insurers, we used the theory of VaR to derive a closed-form formula for an MI premium when MI insurers seek to restrain the systematic risk at a given confidence level.

The numerical analyses yielded several important findings. First, all the estimates of β for the CAPM for the 51 U.S. states are significant at the 1% level when the U.S. HPI was adopted as the proxy for market portfolio, but those estimates are significant only for 4 states, and no β is significant at the 1% level, when the S&P 500 was adopted as the proxy for market portfolio. This result is consistent with the argument from previous studies, namely, that the U.S. HPI has much more explanatory power than the S&P 500 Index. Second, our estimates of β using the U.S. HPI range from 0.2106 to 2.5173. These values are within the scope of the estimated β values found in previous studies. Third, our results show that the idiosyncratic risk is a larger proportion of the total housing risk than is the systematic risk. Therefore, one can infer that most of the effect of the total housing risk may be ascribed to the idiosyncratic risk.

Fourth, our results reveal that the MI premiums calculated by our model considering both systematic and idiosyncratic risks are higher than those calculated by the models that do not consider these two risks. The average values for MI^T and MI_{HPI}^S are 0.0707% and 2.4576%, respectively. Given the model's parameters, our numerical results seem more reasonable because numerous MI premiums calculated by the traditional method appear quite unreasonable. Fifth, our results reveal that the MI premiums indeed went up because of the increase in the systematic risk caused by the 2007 financial crisis, a fact demonstrated regardless of which model is used to calculate the MI premiums.

Finally, summarizing the findings of our sensitivity analyses, we can infer that the MI premium (MI_{HPI}^S) is negatively related to α but positively related to the parameters of the systematic risk (β and σ_M) and the idiosyncratic risk (σ_ε). Thus, when MI insurers seek to fairly price an MI premium and undertake risk management, both of the systematic and idiosyncratic risks should be considered in the valuation model. Accordingly, our analyses should help MI insurers adjust MI premiums under various economic conditions.

To the best of our knowledge, our pricing model is the first to provide a closed-form formula for MI valuation considering both systematic and idiosyncratic risks. Moreover, our model can be used to calculate MI premiums considering systematic risk at different confidence levels. Our pricing formula should make it easier for MI insurers to take account of the economic environment in determining fair MI premiums. Until now, the MI premiums of both the FHA and PM depend on the terms of the mortgage or the borrower's credit, but these two risks have not been considered. From the insurer's perspective, the understanding of their exposure to systematic and to idiosyncratic risks can help them modulate MI premiums in response to changing economic conditions. If they can obtain MI premiums protected against the losses caused by the risks, they may need fewer extra reserve funds to satisfy the solvency capital requirements, and thus they can efficiently allocate the capital in relation to the minimum reserve and capital requirements.

In general, the mortgagors pay the MI premium to MI companies. If MI company receive the MI premium without considering the PCR of systematic risk. MI companies may incure extra costs caused from the requirement for the PCR of systematic risk. Thus, MI companies need to consider the hedging problem by portfolio theory. Nowadays, most of the models of MI premium do not consider the PCR of systematic risk. Thus, if using such model to evaluate MI premiums, to discuss the management of MI portfolio on the hedging of systematic risk should be another interesting issue. In addition, to consider the possible early default probability for MI, our model follows the specifications in Bardhan et, al. (2006) that assumes the default probability is an exogenous in the model. However, in fact, the default probability may be influenced by house price. Based on such specification, the MI premium is modeled by the American-type option. This should be an interesting issue in the future study.

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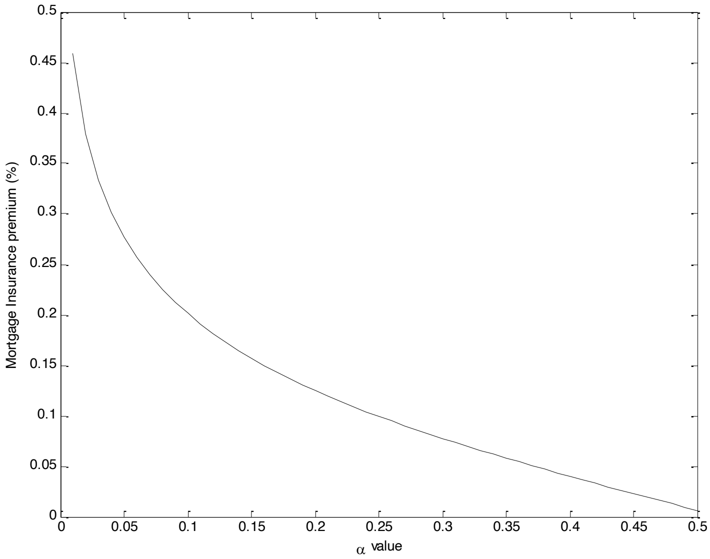
Appendix

This appendix gives the code names of the U.S. 51 states, including the District of Columbia.

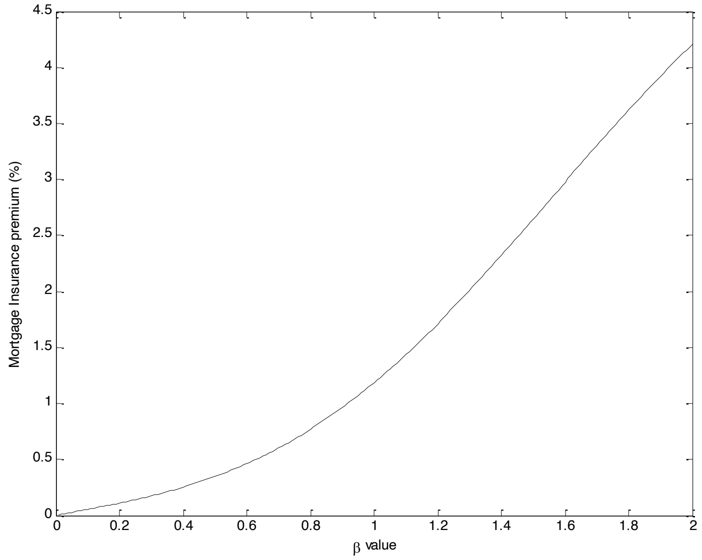
Code Name	State	Code Name	State	Code Name	State
AK	ALASKA	KY	KENTUCKY	NY	NEW YORK
AL	ALABAMA	LA	LOUISIANA	OH	OHIO
AR	ARKANSAS	MA	MASSACHUSETTS	OK	OKLAHOMA
AZ	ARIZONA	MD	MARYLAND	OR	OREGON
CA	CALIFORNIA	ME	MAINE	PA	PENNSYLVANIA
CO	COLORADO	MI	MICHIGAN	RI	RHODE ISLAND
CT	CONNECTICUT	MN	MINNESOTA	SC	SOUTH CAROLINA
DC	DISTRICT OF COLUMBIA	MO	MISSOURI	SD	SOUTH DAKOTA
DE	DELAWARE	MS	MISSISSIPPI	TN	TENNESSEE
FL	FLORIDA	MT	MONTANA	TX	TEXAS
GA	GEORGIA	NC	NORTH CAROLINA	UT	UTAH
HI	HAWAII	ND	NORTH DAKOTA	VA	VIRGINIA
IA	IOWA	NE	NEBRASKA	VT	VERMONT
ID	IDAHO	NH	NEW HAMPSHIRE	WA	WASHINGTON
IL	ILLINOIS	NJ	NEW JERSEY	WI	WISCONSIN
IN	INDIANA	NM	NEW MEXICO	WV	WEST VIRGINIA
KS	KANSAS	NV	NEVADA	WY	WYOMING

Figure 1: Sensitivity analyses for MI premiums corresponding to changes in the confidence level (α)



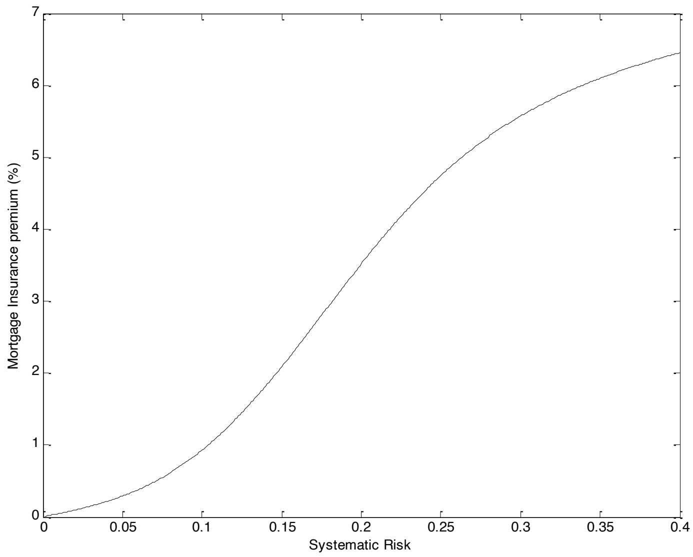
Note: The y-axis represents MI premiums. The x-axis represents confidence level α , which ranges from 0.01 to 0.5.

Figure 2: Sensitivity analyses for MI premiums corresponding to changes in β



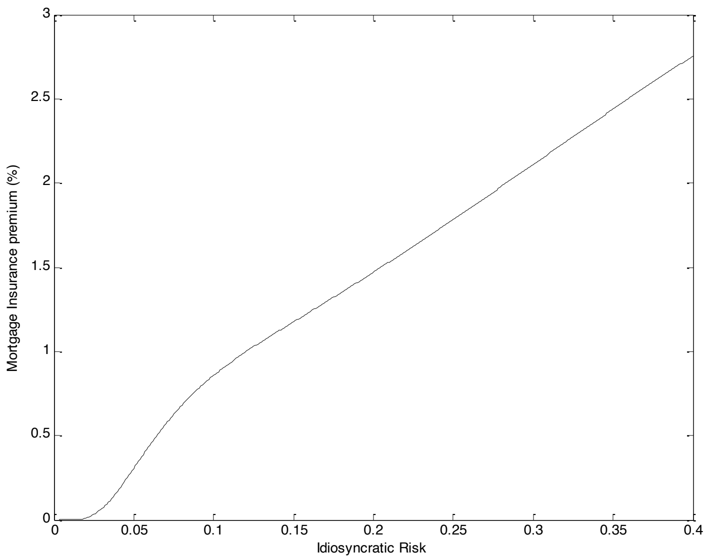
Note: The y-axis represents MI premiums. The x-axis represents β , which ranges from 0 to 2.

Figure 3: Sensitivity analyses for MI premiums corresponding to changes in the systematic risk



Note: The y-axis represents MI premiums. The x-axis represents σ_M, which ranges from 0 to 0.4.

Figure 4: Sensitivity analyses for MI premiums corresponding to changes in the idiosyncratic risks



Note: The y-axis represents MI premiums. The x-axis represents σ_ε, which ranges from 0 to 0.4.

Table 1: Statistical summary for the returns of housing price indices and the S&P 500 index

	State HPI	U.S. HPI	S&P 500
Mean	0.0358	0.0372	0.0866
Std	0.0464	0.0506	0.1682
Max	0.1059	0.3387	0.4657
Median	0.0461	0.0379	0.1032
Min	-0.1007	-0.3164	-0.3968

Note: This table gives the statistical summary for the returns of U.S. HPI, all state HPIs, and the S&P 500 Index. "Mean", "Std", "Max", "Med", and "Min" stand for the mean, standard deviation, minimum, median, and maximum, respectively. The S&P 500 Index, U.S. HPI and state HPI data are all quarterly data from 1991Q1 to 2017Q2. All data for the U.S. HPI and state HPIs were obtained from the Federal Housing Finance Agency (FHFA).

Table 2: Summary of default probabilities for each calendar year

Calendar Years	Default probability	Calendar Years	Default probability
1	0.0003%	16	0.6703%
2	0.0157%	17	0.3297%
3	0.1350%	18	0.3720%
4	0.3183%	19	0.5067%
5	0.3837%	20	0.6993%
6	0.3997%	21	0.7533%
7	0.3897%	22	0.8880%
8	0.3687%	23	0.7470%
9	0.6230%	24	0.5990%
10	0.8437%	25	0.8857%
11	0.9633%	26	1.5660%
12	0.5283%	27	1.6377%
13	0.5823%	28	1.4737%
14	0.7140%	29	2.5890%
15	0.6393%	30	1.6900%

Note: The data regarding the prepayment and default probabilities were obtained from the Department of Housing & Urban Development’s 2010 FHA annual actuarial report. These reports were sampled yearly from 1998 to 2010. This table shows the default probabilities that were obtained by calculating the means for each calendar year the mortgage was issued.

Table 3: Estimated parameter values for the CAPM, systematic volatility (σ_M) and idiosyncratic volatility (σ_ε)

Market Portfolio	κ	β	σ_H	σ_M	σ_ε
U.S. HPI	0.0207***	0.4260***	0.0521	0.0482	0.0479
	(0.0005)	(0.0000)			
S&P 500	0.0376***	-0.0261	0.0521	0.3013	0.0515
	(0.0000)	(0.1246)			

Note: In this table, we use only one state (Alaska) as an example to show the estimates. We respectively adopted the U.S. HPI and S&P 500 Index as the market portfolio to estimate the intercept term κ and β in the CAPM (Equation (11)). σ_H represents the housing risk for each state. σ_M represents the risk for market portfolio, σ_ε represents the idiosyncratic housing risk, estimated by the CAPM. The value in the parenthesis is the P-value. *** denotes significance at the 1 percent level.

Table 4: Estimates of beta for all the states (market portfolio adopted from the U.S. HPI)

State	β	P-value	State	β	P-value	State	β	P-value
AK	0.4260***	0.0000	**KY**	0.3935***	0.0000	**NY**	0.7932***	0.0000
AL	0.5760***	0.0000	**LA**	0.4571***	0.0000	**OH**	0.5778***	0.0000
AR	0.4955***	0.0000	**MA**	0.9457***	0.0000	**OK**	0.3699***	0.0000
AZ	2.0660***	0.0000	**MD**	1.4173***	0.0000	**OR**	1.1783***	0.0000
CA	2.2601***	0.0000	**ME**	0.8755***	0.0000	**PA**	0.6910***	0.0000
CO	0.6895***	0.0000	**MI**	0.9179***	0.0000	**RI**	1.2875***	0.0000
CT	0.9194***	0.0000	**MN**	1.0255***	0.0000	**SC**	0.6966***	0.0000
DC	1.1639***	0.0001	**MO**	0.6506***	0.0000	**SD**	0.3213***	0.0008
DE	1.0117***	0.0000	**MS**	0.4453***	0.0000	**TN**	0.5998***	0.0000
FL	2.0860***	0.0000	**MT**	0.6248***	0.0000	**TX**	0.4135***	0.0000
GA	0.9922***	0.0000	**NC**	0.6139***	0.0000	**UT**	0.8932***	0.0000
HI	1.5085***	0.0000	**ND**	0.2106***	0.0269	**VA**	1.1067***	0.0000
IA	0.3340***	0.0000	**NE**	0.3665***	0.0000	**VT**	0.6220***	0.0001
ID	1.1588***	0.0000	**NH**	1.0809***	0.0000	**WA**	1.1749***	0.0000
IL	0.8159***	0.0000	**NJ**	1.1117***	0.0000	**WI**	0.6189***	0.0000
IN	0.4206***	0.0000	**NM**	0.6725***	0.0000	**WV**	0.3068***	0.0185
KS	0.4367***	0.0000	**NV**	2.5173***	0.0000	**WY**	0.4593***	0.0002

Note: This table summarizes the β estimates for each U.S. state when we use the U.S. HPI as the market portfolio. For the definitions of the code names of the states, see Appendix. *** denotes significance at the 1 percent level.

Table 5: Estimates of beta for all the states (market portfolio adopted from the S&P 500 index)

State	β	P-value	State	β	P-value	State	β	P-value
AK	-0.0261	0.1246	**KY**	0.0185*	0.0704	**NY**	-0.0060	0.7267
AL	0.0025	0.8772	**LA**	0.0067	0.5983	**OH**	0.0251**	0.0306
AR	-0.0092	0.5924	**MA**	-0.0084	0.6847	**OK**	0.0177	0.1856
AZ	0.0584	0.1134	**MD**	0.0048	0.8556	**OR**	0.0046	0.8460
CA	0.0669*	0.0907	**ME**	-0.0148	0.5547	**PA**	0.0088	0.5170
CO	0.0096	0.5807	**MI**	0.0140	0.5060	**RI**	-0.0245	0.4098
CT	-0.0105	0.5898	**MN**	0.0020	0.9186	**SC**	0.0106	0.5393
DC	0.0583	0.2392	**MO**	-0.0017	0.8982	**SD**	0.0026	0.8712
DE	-0.0009	0.9696	**MS**	0.0145	0.3599	**TN**	0.0043	0.7440
FL	0.0534	0.1233	**MT**	-0.0017	0.9358	**TX**	0.0103	0.3028
GA	0.0158	0.4342	**NC**	0.0034	0.8191	**UT**	0.0390	0.1070
HI	-0.0541	0.2276	**ND**	0.0069	0.6527	**VA**	0.0072	0.7203
IA	0.0148	0.1692	**NE**	0.0221*	0.0978	**VT**	0.0214	0.4147
ID	-0.0090	0.7298	**NH**	-0.0286	0.2477	**WA**	0.0216	0.3327
IL	0.0092	0.5493	**NJ**	-0.0153	0.4703	**WI**	-0.0018	0.8956
IN	0.0160	0.1271	**NM**	0.0035	0.8554	**WV**	0.0063	0.7642
KS	-0.0007	0.9592	**NV**	0.0709	0.1086	**WY**	0.0060	0.7681

Note: This table summarizes the β estimates for each U.S. state when we use the S&P 500 as the market portfolio. For the definitions of the code names of the states, see Appendix. * and ** denote significance at the 10 and 5 percent levels respectively.

Table 6: Ratios of the idiosyncratic risk to the whole housing risk for each state (market portfolio adopted from the U.S. HPI)

State	σ_H	σ_ε	Ratio	State	σ_H	σ_ε	Ratio	State	σ_M	σ_ε	ratio
AK	0.0521	0.0479	91.92%	**KY**	0.0315	0.0251	79.78%	**NY**	0.0522	0.0356	68.14%
AL	0.0488	0.0401	82.22%	**LA**	0.0389	0.0321	82.46%	**OH**	0.0358	0.0224	62.74%
AR	0.0525	0.0468	89.07%	**MA**	0.0629	0.0434	68.98%	**OK**	0.0410	0.0369	90.05%
AZ	0.1131	0.0537	47.45%	**MD**	0.0805	0.0426	52.91%	**OR**	0.0727	0.0455	62.50%
CA	0.1214	0.0537	44.22%	**ME**	0.0764	0.0637	83.37%	**PA**	0.0416	0.0250	60.03%
CO	0.0532	0.0415	78.07%	**MI**	0.0642	0.0466	72.52%	**RI**	0.0909	0.0664	73.09%
CT	0.0593	0.0394	66.50%	**MN**	0.0582	0.0308	52.93%	**SC**	0.0528	0.0408	77.23%
DC	0.1515	0.1408	92.90%	**MO**	0.0395	0.0241	60.89%	**SD**	0.0480	0.0454	94.65%
DE	0.0752	0.0573	76.15%	**MS**	0.0484	0.0434	89.63%	**TN**	0.0399	0.0275	68.90%
FL	0.1064	0.0348	32.72%	**MT**	0.0631	0.0555	87.89%	**TX**	0.0306	0.0232	75.87%
GA	0.0616	0.0388	62.98%	**NC**	0.0455	0.0346	75.99%	**UT**	0.0742	0.0605	81.46%
HI	0.1372	0.1164	84.81%	**ND**	0.0470	0.0459	97.64%	**VA**	0.0614	0.0304	49.47%
IA	0.0330	0.0288	87.29%	**NE**	0.0409	0.0369	90.22%	**VT**	0.0800	0.0741	92.71%
ID	0.0799	0.0571	71.51%	**NH**	0.0757	0.0549	72.56%	**WA**	0.0683	0.0381	55.86%
IL	0.0469	0.0255	54.44%	**NJ**	0.0646	0.0360	55.81%	**WI**	0.0415	0.0288	69.52%
IN	0.0322	0.0250	77.71%	**NM**	0.0593	0.0497	83.74%	**WV**	0.0644	0.0627	97.33%
KS	0.0390	0.0328	84.18%	**NV**	0.1355	0.0605	44.61%	**WY**	0.0622	0.0581	93.45%

Note: For the definitions of the code names of the states, see Appendix. The ratio equals σ_ε divided by σ_H. Other definitions see Table 3.

Table 7: MI premiums for each state estimated by the traditional model and our model

State	MI^T	MI^S_{HPI}	State	MI^T	MI^S_{HPI}	State	MI^T	MI^S_{HPI}
AK	0.0048	0.2767	**KY**	0.0001	0.0257	**NY**	0.0048	0.5012
AL	0.0032	0.3004	**LA**	0.0007	0.1006	**OH**	0.0003	0.0642
AR	0.0050	0.3264	**MA**	0.0134	0.9683	**OK**	0.0010	0.1058
AZ	0.1452	4.3429	**MD**	0.0414	2.3341	**OR**	0.0267	1.6099
CA	0.1821	4.8109	**ME**	0.0331	1.2196	**PA**	0.0011	0.1738
CO	0.0054	0.4629	**MI**	0.0148	0.9742	**RI**	0.0672	2.2073
CT	0.0098	0.8249	**MN**	0.0089	0.8913	**SC**	0.0052	0.4593
DC	0.3472	2.5482	**MO**	0.0008	0.1263	**SD**	0.0029	0.1647
DE	0.0309	1.3933	**MS**	0.0030	0.2306	**TN**	0.0008	0.1331
FL	0.1183	4.5657	**MT**	0.0136	0.6339	**TX**	0.0001	0.0207
GA	0.0119	0.9774	**NC**	0.0020	0.2483	**UT**	0.0292	1.1965
HI	0.2628	3.0995	**ND**	0.0025	0.1029	**VA**	0.0117	1.1126
IA	0.0002	0.0325	**NE**	0.0010	0.1043	**VT**	0.0403	0.9287
ID	0.0401	1.7482	**NH**	0.0318	1.5154	**WA**	0.0199	1.4719
IL	0.0025	0.3366	**NJ**	0.0152	1.2414	**WI**	0.0011	0.1662
IN	0.0001	0.0311	**NM**	0.0098	0.5961	**WV**	0.0150	0.3348
KS	0.0007	0.0977	**NV**	0.2537	5.2733	**WY**	0.0126	0.4567

Note: The unit is % in this table. MI^T represents the MI premium calculated by the traditional model (Bardhan et al., 2006). MI^T is calculated by Equation (10). MI^S_{HPI} represents the MI premium calculated by our model when we use the U.S. HPI as the market portfolio. MI^S_{HPI} is calculated by Equation (34). The basic parameters for calculating MI (i.e., MI^T and MI^S_{HPI}) are: $T = 30$ years, c = 3%, r = 2%, η = 95%, $H(0)$ = \$100 thousand, $M(0)$ = \$95 thousand, δ = 1%, l_R = 75%. β and σ_ε are shown in Tables 4 and 6, respectively.

Table 8: Statistical summary for the estimated MI premiums

	MI^T	MI^S_{HPI}
Mean	0.0364	1.0562
Std	0.0736	1.3080
Max	0.3472	5.2733
Median	0.0098	0.5012
Min	0.0001	0.0207

Note: The unit is % in this table. The means and standard deviations of MI (i.e., MI^T and MI^S_{HPI}) are calculated from the MIs for each of the 51 states. For other definitions, see Tables 1 and 7.

Table 9: Statistical summary for the estimated MI premiums with respect to the 2007 financial crisis

Period		r_{HPI}	MI^T	MI^S_{HPI}
Before Financial Crisis	Mean	5.2604	0.0209	0.6207
	Std	2.7852	0.0637	1.0236
After Financial Crisis	Mean	0.9719	0.0543	1.3368
	Std	5.8659	0.1014	1.5859

Note: The unit is % in this table. r_{HPI} denotes the yearly return for the U.S. HPI. We divide the sample period into two sub-periods: Before Financial Crisis and After Financial Crisis. Before Financial Crisis is defined as the period from 1991Q2 to 2007Q1; After Financial Crisis is defined as the period from 2007Q2 to 2017Q2. The means and standard deviations of r_{HPI} are calculated from the data in the subsamples. The means and standard deviations of MI are calculated from the estimated MIs for each of the 51 states. For other definitions, see Tables 1 and 7.